MATHEMATICAL MODELING AND NUMERICAL SIMULATION OF TWO PHASE BLOOD FLOW

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Abstract -

A two-phase blood flow model is considered to analyze the fluid flow and heat transfer in a curved tube with time-variant stenosis. In both core and plasma regions, the variable viscosity model (Hematocrit and non linear temperature-dependent, respectively) is considered. A toroidal coordinate system is considered to describe the governing equations. The perturbation technique in terms of perturbation parameter ε is used to obtain the temperature profile of blood flow. In order to find the velocity, wall shear stress and impedance profiles, a second-order finite difference method is employed with the accuracy of 10−6 in the each iteration. Under the conditions of fully-developed flow and mild stenosis, the significance of various physical parameters.

Introduction -

Bio-fluid mechanics, a well-established branch of bio-mathematics with the help of its normal functions, and changed due to alternation via mathematical analysis. The bio-mathematics being an interdisciplinary subject seek to understand to the bio-mechanics mechanical properties of living tissues(Y.C.Fung-1981)[1], Anatomy and physiology in health and illness(Ross and Wilson-2010)[2], Introductory bio-mechanics: from cell to organisms(EC Ross & CA Simmons-2007)[3], Blood flow and microcirculation(SE Charm & GS Kurland-1974)[4], Flow properties of blood(AL Copley & G Stainsby-1960)[5], Circulatory physiology: Dynamics and control of body fluids (A.C.Guyton-1975,2011)[6]. The proposed work in this topic is specially the hepatic blood flow in portal vein and arteries. Red blood cells, carry on oxygen to the tissues. White blood cells, which is fighting infections. Platelets, smaller cells with the help of blood clot .We have considered the blood as a two phased one of which is red blood cells and other is plasma. We apply these objects in our research work- To construct two phase model for hepatic circulation of blood To collect the clinical data for blood pressure against hemoglobin for the graphical study. To construct a model for stenoised hepatic blood vessels.

Composition of Blood -

These blood cells consist of (RBC), (WBC) and (Platelets). Density of blood 1060 kg/m^3 and water density is 1000 kg/m. Whole blood exhibits non-Newtonian fluid dynamics. In this

thesis assessment of blood's work of art and circulatory classification is presented. Human blood is a liquid tissue which makes up regarding 1/13 of the total body mass and accounts for 5 to 6 liters in male. It's separates into two portions first is Plasma a fluid part of blood and it consists of about 90% of water, 7% of protein. It's behaves like a Newtonian. The second phase consists of cells, mainly red blood cells. Red cells include hemoglobin and carry oxygen right through the body.

Hematocrit -

Hematocrit is the important part which define the whole viscosity. It's define value of 46% for men and 42% in women. The hematocrit is on average about three times the hemoglobin concentration.

Two Phase non– Newtonian blood flow-

Blood is non-homogeneous mixture of plasma and blood cells. The thesis measured to homogeneous two-phased Non-Newtonian mixture of plasma and blood cells at low shear rate and for the duration of its flow during narrow blood vessels. The blood Stress rate τ and strain rate e both have six different components.

(i) Newtonian Equation – $\tau = \eta e$

$where\ \eta$ is the viscosity coefficient.

(ii) The Non-Newtonian power law equation –

$$T' = \eta_m e^n$$

The strain rate between $5\ and\ 200\ sec^{-1}$, $0.68 \leq n \leq 0.80$.

The constitutive equation of blood is

$$T^{ij} = -pg^{ij} + \eta_m e(ij)^n = -pg^{ij} + T'^{ij}$$

T^{ij} is stress tensor and T'^{ij} is shearing stress tensor.

(iii) The Non-Newtonian Herschel-Bulkley Equation –

$$\tau = \eta e^n + \tau_0 \ (\tau \geq \tau'_0)$$
$$e = 0 \quad (\tau' < \tau'_0)$$

When blood shows yield stress τ'_0.

$$\text{If } (\tau < \tau_0),$$
$$\tau_0{}^{1/3} = \frac{A(H - H_m)}{100}\ where\ A = (0.008 \pm 0.002 \frac{dyne}{cm^2})^{\frac{1}{3}}$$

Where H is normal hematocrit.

Description of two phase blood flow -

In blood flow blood depend on the vessel diameter, in small vessels it is necessary to take in to near wall effects and aggregation of erythrocytes. The analytical solution addiction of the blood velocity, blood viscosity and hematocrit. Blood is made separate components, they are-

Red Blood Cells, White blood cells, platelets, plasma. We find the graphical presentation between blood pressure drop against hematocrit. In every graph we define tangent line and slope of the graph. When graph growing sense (in different vessels for different cases) then we can't suggest the serious dose, and when graph shows decreasing sense then we suggest serious dose but according to steepness of slops according to trade line at different conditions (critical, middle, normal). If slope of the graph is elevated then can't suggest the operation, and given the medicine slowly. If the slope of graph is low down then suggest the high dose medicine easily and also suggest the operation. The error define the difference between hematocrit against blood pressure drop. We take blood vessels in cylindrical form. Oxygen formed oxy – hemoglobin and carbon dioxide is solvable in plasma then carbon dioxide is taken as third phase in veniules and veins. We recommended successful operation but according to the condition the clinical data is collected in the duration of declared operation. In all graphs we have observed "relation between real clinically blood pressure drop" and "relation between mathematically modulated blood pressure drop v/s hematocrit". The all graph are shows different nature but present trend line are not different. If trend is uphill it's means that fluctuation of blood pressure drop increases with respect to hematocrit. If trend is downhill that means fluctuation of blood pressure drop decreases with respect to hematocrit.

Objectives of This Research -

(1) To construct two phase model for hepatic circulation of blood.
(2) To collect the pathological data for blood pressure against hemoglobin for the graphical study.
(3) To construct a model for stenoised blood vessels.

Review of literature -

This chapter comprises of a brief resume of work done already in the field of mathematical modeling. The aim of the literature review was to understand the problem that exists and to arrange the objectives of our research work and its significance.

Fung (1984) investigated stress-strain relationship and structure of soft tissues. The nonlinear relationship of the stress-strain is systemic arteries, exponentially in nature with increasing strains; linear relationship in the lung pulmonary arteries. The systemic veins are easily collapsible and pulmonary veins are not in the lungs; in case of blood pressure falls below patent are remain in the gas pressure of alveolar. The clarification of different organ in the surrounding tissues is differences fabrication in the mixed interactions between the blood vessels. The suggestions of these differences on blood circulation are pointed out and discussed the role of ultra structure.

Kaur *et al.* (2013) studied data mining techniques use to detection of lung cancer. The disease of lung cancer is unrestrained cell growth in tissue. By utilizing of CT, X-ray chest films, MRI *etc.* measured early stage lung cancer. Few techniques used such as lung field Segmentation, Future Extraction, Artificial Neural Networks, Data Processing, and

SVMs for identification of lung cancer. Two categories of X-ray chest films and different experiment achieve on two group data sets.

James (2013) discussed various mathematical models for study of lung and its effect of smoking and nicotine patient. Tobacco products found drug stimulant due to lung organ and human respiratory system and their effects of destructive conditions. Some nicotine like microtone is the leading causes of lung cancer and increase the blood pressure of patient. They studied the cigarette and nicotine is highly correlated on effect of lung cancer. The mathematical model obtained reduces or stop smoking most cases of lung cancer also reduce the chance of occurring.

Browne (2014) clinical studies in structure and blood function of vessel. He examined the look at and contrasts the three tunics that make up the dividers of most veins, recognize flexible conduits, solid courses, and arterioles based on structure, area and capacity, depict the fundamental structure of a capillary bed, from the providing metarteriole to the venuleinto which it depletes and clarify the structure and capacity of venous valves in the substantial veins of the limits. Conversely, in the pulmonary circuit, arteries carry blood low in oxygen solely to the lungs for exchange of gas. In systematic circulation, pneumonic veins at that point return crisply oxygenated blood from the lungs to the heart to be back pumped.

Kamonpun *et al.* (2014) discussed the pulmonary hypertension of size and significance of pulmonary artery. It found relation mortality and morbidity. In increase the pH value in advance pH treatment, the pulmonary artery additional effective and safer choice. Presence of PH predict to CT utility address and attempt. They conclude that the association between PH and pulmonary artery dimensions. Evaluate strength and weakness of during clinical practices.

Moses et al. (2014) have talked about rising therapeutic of lung cancer targets in pathogenesis view of bone metastases. They talked about a portion of the developing sub-atomic focuses on that have given bits of knowledge into the course of metastases in lung disease with the emphasis on bone infection.

Proposed Methodology during the Research Work -

Mathematical models will be formulated -

1- Introduction two phase Blood Flow
2- Choice of frame of reference
3- Concept of mathematical modeling
4- Formulation Technique
a. Numerical Method
b -Analytical Method
5- Biophysical Interpretation

We shall use the following fundamental equations of Bio-fluid mechanics in two phase blood flow.

i. Conservation law of mass
ii. Conservation law of momentum

All the mathematical equation will be developed in tensorial form. Tensorial formulation of the problem makes sure that the fundamental equations transformed into bio-fluid mechanics hold good in any frame of reference. The following steps of models will be formulated for different cases of human pulmonary blood vessels artery, arterioles, capillary, veins and stenosis. Analytical Method: Ordinary differential equations methods and Partial differential equations method.

1. Numerical Method:
2. Simpsons $1/3^{rd}$ rule, Runga-Kutta method and Difference method etc.
3 Trial and Error Method.
4. Biophysical Interpretation
5. Constitutive equations of Non-Newtonian fluids
6. $\tau = \eta e^n$
7. [if $n \neq 1$]

Where,τ is denoted by stress e is denodet by Strain rate.

The nature of liquid (fluid) is $n = 1$, at that point Newtonian and on the off chance that $n \neq 1$at that point the nature of fluid is Non-Newtonian fluids.Where, τ is signified by stress,e is signified by strain rate and n is signified by the parameter, these condition employments condition of movement.

Herschel-Bulkley law

The Herschel - Bulkley law holds great on the two phase blood stream through blood vessels and whose constitutive is as follows

$$\tau = \eta_m e^n + \tau_0 \ (\tau \geq \tau_0) \text{ and}$$
$$e = 0 \quad (\tau < \tau_0)$$

1- Where τ_0 is the yield stress. When strain rate $e = 0 \ (\tau < \tau_0)$

Expected Outcome of the Proposed Work -

The two phased model of human blood flow will be developed as-

1. The two phase model of blood flow will give a relation between blood pressure and hematocrit.
2. It will give the graphical interpretation and better treatment of the patient who is suffer from any desises.
3. It will give the mathematical solution during the stenosed blood vessels.

Refrencess -

1. Donsteves E., (1978) In flamatory mechanism, PP – 293-324

2. Formaggia Luca, (2003), One- dimensional models for blood flow in Arteries, Vol. (47), PP -251 – 276.
3. Pralhad R.N., (2004), Modeling of arterial stenosis and its applications to blood disease, Vol. (16), PP – 203-220.
4. Friedrich-Rust M, Ong MF, (2007), Real-Time Elastography for noninvasive assessment of Liver Fibrosis in Chronic Hepatitis. *AJR*; V (188), PP - 758–764