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q-Fractional Integral Inequalities Involving Saigo's Operator

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Abstract

The aim of this present paper is to prove some novel fractional integral inequalities for synchronous functions connected to the Chebyshev functional, involving the Gauss hypergeometric function and presents a number of special instances as fractional integral inequalities involving Riemann-Liouville type fractional integral operators. Additionally, we take into account their applicability to other relevant, previous findings.
Keywords: Fractional Integral Inequalities, Saigo's Operator and q-Saigo's Operator.

MSC: Primary 26A33; Secondary 26D10, 05A30

1 Introduction

The most beneficial uses of fractional integral inequalities are in determining the uniqueness of solutions to fractional boundary value issues and fractional partial differential equations. Additionally, they offer upper and lower bounds for the solutions of the aforementioned equations. These factors have prompted a number of scholars working in the area of integral inequalities to investigate various extensions and generalizations by utilizing fractional calculus operators. For instance, the book [1] and the publications [2-11] both contain references to such works. Purohit and Raina [9] recently looked into some integral inequalities of the Chebyshev type [12] utilizing Saigo fractional integral operators and established the q-extensions of the main findings. Present study uses the fractional hypergeometric operator proposed by Curiel and Galue [13] to prove a few generalized integral inequalities for synchronous functions related to the Chebyshev functional. As special examples of our findings, the results attributed to Purohit and Raina [9] and Belarbi and Dahmani [2] are presented below.

2 PRELIMINARIES:

Definition 1: Two functions f and g are said to be synchronous on the interval [a,b] if:

$$\Big(f(x)-f(y)\Big)\Big(g(x)-g(y)\Big)\geq 0, \quad for \quad any \quad x,y\in[a,b] \tag{1}$$

Definition 2: A real-valued function $f(t)$ $(t > 0)$ is said to be in the space $C_\mu(\mu \in R)$ if there exists a real number $p > \mu$, such that $f(t) = t^p\Phi(t)$,where $\Phi(t) \in C(0,\infty)$.

Definition 3:
Saigo's Fractional Operator: Useful and interesting generalization of both the Riemann-Liouville and Erdelyi- Kober fractional integration operators is introduced by Saigo [13] in terms of Gauss's hypergeometric function as given below:
Let α, β and η are complex numbers and let $x \in R+$ the fractional $Re(\alpha) > 0$ and the fractional derivative $Re(\alpha) < 0$ of the first kind of a function

$$I_{0,x}^{\alpha,\beta,\eta} f(x) = \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} {}_2F_1(\alpha+\beta, -\eta; \alpha; 1-\frac{t}{x}) f(t)dt$$
$$= \frac{d^n}{dx^n} I_{0,x}^{\alpha+n,\beta+n,\eta-n} f(x), 0 < Re(\alpha) + \eta \leq 1 (n \in N_0).$$

$$I_{0,x}^{\alpha,\beta,\eta,\mu} f(x) = \frac{x^{-\alpha-\beta-2\mu}}{\Gamma(\alpha)} \int_0^x t^\mu (x-t)^{\alpha-1} {}_2F_1(\alpha+\beta, -\eta; \alpha; 1-\frac{t}{x}) f(t)dt \qquad (2)$$

2.1 q-Extension:

Recently, q-calculus has served as a bridge between mathematics and physics. Consequently, there has been a significant increase in activities in the field of q-calculus due to applications of q-calculus in mathematics, statistics, and physics. Most of the scientists in the world who use the q-calculation today are physicists. The q-calculus is a generalization of many topics, such as chain metaphysics, generating functions, complexity analysis, and particle physics. In short, q-calculation is a very popular topic today. One of the important branches of q-calculus in number theory is the q-class of special generating functions, such as q-Bernoulli numbers, q-Euler numbers and q-Genocchi numbers. Here we define a new class of multi-variable integrals of type q (generalizations of beta integrals) called Saigo's q-Integral Operator.
The Saigo's q-Integral operator which is analogue to the well known Bernardi integral operator is investigated. we obtain a simple conceptual q-extension of the results obtained in the first part of the paper, using some q-Saigo's q-Integral operator in the next section.

Definition 4:
Saigo's q-Integral Operator:
A basic analogue of Saigo's fractional integral operator [14] is defined as

$$I_q^{\alpha,\beta,\eta} f(x) = \frac{x^{-\beta-1}}{\Gamma_q(\alpha)} \int_0^x (\frac{tq}{x};q)_{\alpha-1} \sum_{m=0}^{\infty} \frac{(q^{\alpha+\beta},q)_m (q^{-\eta},q)_m}{(q^\alpha,q)_m (q;q)_m} q^{(\eta-\beta)m} (-1)^m (q)^{-\binom{m}{2}} (t/x-1)_m f(t) d_q t$$

and

$$f(x) = \frac{q^{-\binom{\alpha}{2}-\beta}}{\Gamma_q(\alpha)} \int_0^x (\frac{x}{t};q)_{\alpha-1} \sum_{m=0}^{\infty} \frac{(q^{\alpha+\beta},q)_m (q^{-\eta},q)_m}{(q^\alpha,q)_m (q;q)_m} q^{(\eta-\beta)m} (-1)^m (q)^{-\binom{m}{2}} (x/qt-1)_m f(tq^{1-\alpha}) d_q t$$

by using q-integral definition, the above operators can be written as

$$I_q^{\alpha,\beta,\eta} f(x) = x^{-\beta}(1-q)^\alpha \sum_{m=0}^{\infty} \frac{(q^{\alpha+\beta},q)_m (q^{-\eta},q)_m}{(q;q)_m} q^{(\eta-\beta+1)m} \sum_{k=0}^{\infty} q^k \frac{(q^{\alpha+m},q)_k}{(q;q)_k} f(xq^{k+m})$$

and

$$K_q^{\alpha,\beta,\eta} f = x^{-\beta} q^{-\binom{\alpha}{2}} (1-q)^\alpha \sum_{m=0}^{\infty} \frac{(q^{\alpha+\beta},q)_m (q^{-\eta},q)_m}{(q;q)_m} q^{\eta m} \sum_{k=0}^{\infty} q^{\beta k} \frac{(q^{\alpha+m},q)_k}{(q;q)_k} f(xq^{-\alpha-k-m})$$

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3 Main Results:

3.1 *In this section we obtain certain Chebyshev type integral inequalities involving the generalized fractional integral operator. The following lemma is used for our first result.*

Lemma 1: *Let $\alpha > 0, \mu > -1, \beta, \eta \in R$; then the following image formula for the power function under the operator(Def.2) holds true:*

$$I_{0,x}^{\alpha,\beta,\eta}(x^{t-1}) = \frac{\Gamma(t)\Gamma(t-\beta+\eta)}{\Gamma(t-\beta)\Gamma(t+\alpha+\eta)} x^{t-\beta-\mu-1} \tag{3}$$

Theorem 1. *Let f and g be two synchronous functions on $(0, \infty)$ then*

$$I_{0,x}^{\alpha,\beta,\eta}\{f(x)g(x)\} \geq \frac{\Gamma(1-\beta)\Gamma(1+\alpha+\eta)}{\Gamma(1-\beta+\eta)} x^{\beta+\mu} \times I_{0,x}^{\alpha,\beta,\eta}\{f(x)\} I_{0,x}^{\alpha,\beta,\eta}\{g(x)\}$$

For all $x > 0$, $\alpha > max0, -\beta-\mu$, $\beta < 1, \mu > -1, \eta < 0$.
Proof: Let f and g be two synchronous functions, then using definition 1, for all τ, $\rho \in (0,t), t \geq 0$, we have

$$\{(f(\tau) - f(\rho))(g(\tau) - g(\rho))\} \geq 0,$$

Which implies that

$$f(\tau)g(\tau) + f(\rho)g(\rho) \geq f(\tau)g(\rho) + f(\rho)g(\tau) \tag{4}$$

Consider

$$F(x,\tau) = \frac{x^{-\alpha-\beta}(x-\tau)^{\alpha-1}}{\Gamma(\alpha)} \, {}_2F_1\left[\alpha+\beta, -\eta; 1-\frac{t}{x}\right] f(t)dt$$

$$= \frac{(x-\tau)^{\alpha-1}}{\Gamma(\alpha)x^{\alpha+\beta}} \times \frac{(\alpha+\beta)(-\eta)(x-\tau)^{\alpha}}{\Gamma(\alpha+1)x^{\alpha+\beta}} + \frac{(\alpha+\beta+\mu)(\alpha+\beta+1)(x-\tau)^{\alpha-1}}{\Gamma(\alpha+2)} \times \frac{(x-\tau)^{\alpha+1}}{x^{\alpha+\beta+2}} \tag{5}$$

We observe that each term of the above series is positive in view of the conditions stated with Theorem 1, and hence the function $F(x,\tau)$ remains positive, for all $\tau \in (0,t)$.

Multiple both sides of (4) by (5) and integrating w.r.t τ from 0 to t, we get

$$I_{0,x}^{\alpha,\beta,\eta}\{f(x)g(x)\} + f(\rho)g(\rho)I_{0,x}^{\alpha,\beta,\eta}\{1\} \geq g(\rho)I_{0,x}^{\alpha,\beta,\eta}\{f(x)\} + f(\rho)I_{0,x}^{\alpha,\beta,\eta}\{g(x)\} \tag{6}$$

Next, multiplying both sides of (6) by $F(x,\rho)$, $(\rho \in (0,t))$, where $F(x,\rho)$ is given by (5) and integrating w.r.t. ρ from 0 to t, we get the result

$$I_{0,x}^{\alpha,\beta,\eta}\{f(x)g(x)\} \geq \frac{\Gamma(1-\beta)\Gamma(1+\alpha+\eta)}{\Gamma(1-\beta+\eta)} x^{\beta+\mu} \times I_{0,x}^{\alpha,\beta,\eta}\{f(x)\} I_{0,x}^{\alpha,\beta,\eta}\{g(x)\}$$

Theorem 2 *Let f and g be two synchronous functions on $(0, \infty)$ then*

$$\frac{\Gamma(1+\mu)\Gamma(1-\beta+\eta)}{\Gamma(1-\beta)\Gamma(1+\mu+\alpha+\eta)x^{\beta+\mu}} I_{0,x}^{\delta,\gamma,\theta,\zeta}\{f(x)g(x)\} + \frac{\Gamma(1+\theta)\Gamma(1-\delta+\zeta)}{\Gamma(1-\delta)\Gamma(1+\theta+\gamma+\zeta)x^{\beta+\mu}} I_{0,x}^{\alpha,\beta,\eta,\mu}\{f(x)g(x)\}$$

$$\geq I_{0,x}^{\alpha,\beta,\eta,\mu}\{f(x)\} I_{0,x}^{\delta,\gamma,\theta,\zeta}\{g(x)\} + I_{0,x}^{\delta,\gamma,\theta,\zeta}\{g(x)\} I_{0,x}^{\alpha,\beta,\eta,\mu}\{f(x)\}$$

For all $x > 0$, $\alpha > max0, -\beta - \mu$, $\gamma > Max(0, -\delta, -\theta)$, $\mu, \theta > -1$.

Proof: Let f and g be two synchronous functions, then using definition 1, for all τ, $\rho \in (0, t), t \geq 0$, we have

$$\{(f(\tau) - f(\rho))(g(\tau) - g(\rho))\} \geq 0,$$

Which implies that

$$f(\tau)g(\tau) + f(\rho)g(\rho) \geq f(\tau)g(\rho) + f(\rho)g(\tau) \tag{7}$$

Consider

$$F(x, \tau) = \frac{x^{-\gamma-2\theta-\delta}\rho^{\theta}(x-\tau)^{\gamma-1}}{\Gamma(\gamma)}\ {}_2F_1\left[\delta + \delta + \theta, -\zeta; 1 - \frac{\rho}{x}\right] f(\rho)d\rho \tag{8}$$

in light of the circumstances outlined in Theorem 1, we note that each term in the aforementioned series is positive. Multiply both sides of equation (7) by F(x,τ) explained in (8) and integrating w.r.t. τ from 0 to t and using Definition 1, we get

$$I_{0,x}^{\alpha,\beta,\eta,\mu}\{f(x)g(x)\}I_{0,x}^{\delta,\gamma,\theta,\zeta}\{1\} + I_{0,x}^{\delta,\gamma,\theta,\zeta}\{g(x)f(x)\}I_{0,x}^{\alpha,\beta,\eta,\mu}\{1\} \geq$$
$$I_{0,x}^{\alpha,\beta,\eta,\mu}\{f(x)\}I_{0,x}^{\delta,\gamma,\theta,\zeta}\{g(x)\} + I_{0,x}^{\delta,\gamma,\theta,\zeta}\{g(x)\}I_{0,x}^{\alpha,\beta,\eta,\mu}\{f(x)\} \tag{9}$$

Next, multiplying both sides of (9) by $F(x, \rho)$, $(\rho \in (0, t))$, where $F(x, \rho)$ is given by (8) and integrating w.r.t. ρ from 0 to t, we get the result

$$\frac{\Gamma(1+\mu)\Gamma(1-\beta+\eta)}{\Gamma(1-\beta)\Gamma(1+\mu+\alpha+\eta)x^{\beta+\mu}} I_{0,x}^{\delta,\gamma,\theta,\zeta}\{f(x)g(x)\} + \frac{\Gamma(1+\theta)\Gamma(1-\delta+\zeta)}{\Gamma(1-\delta)\Gamma(1+\theta+\gamma+\zeta)x^{\beta+\mu}} I_{0,x}^{\alpha,\beta,\eta,\mu}\{f(x)g(x)\}$$
$$\geq I_{0,x}^{\alpha,\beta,\eta,\mu}\{f(x)\}I_{0,x}^{\delta,\gamma,\theta,\zeta}\{g(x)\} + I_{0,x}^{\delta,\gamma,\theta,\zeta}\{g(x)\}I_{0,x}^{\alpha,\beta,\eta,\mu}\{f(x)\}$$

3.2 *In this section we obtain certain Chebyshev type integral inequalities involving the generalized q-fractional integral operator. The following lemma is used for the below theorem.*

Lemma 2: *Let* $\alpha > 0, \mu > -1, \beta, \eta \in R, q < 1$*; then the following image formula for the power function under the operator(Def.2) holds true:*

$$I_{q,0,x}^{\alpha,\beta,\eta}(x^{\tau-1}) = \frac{\Gamma_q(\tau)\Gamma_q(\tau-\beta+\eta)}{\Gamma_q(\tau-\beta)\Gamma_q(\tau+\alpha+\eta)} x^{\tau-\beta-\mu-1} \tag{10}$$

Theorem 3 *Let f and g be two synchronous functions on* $(0, \infty)$ *then*

$$I_q^{\alpha,\beta,\eta}\{f(x)g(x)\} \geq \frac{\Gamma_q(1-\beta)\Gamma_q(1+\alpha+\eta)}{\Gamma_q(1-\beta+\eta)} x^{\beta+\mu} \times I_q^{\alpha,\beta,\eta}\{f(x)\}I_q^{\alpha,\beta,\eta}\{g(x)\}$$

For all $x > 0$, $\alpha > max0, -\beta - \mu$, $\beta < 1, \mu > -1, \eta < 0, q < 1$.

Proof: Let f and g be two synchronous functions, then using definition 1, for all τ, $\rho \in (0, t), t \geq 0$, we have

$$\{(f(\tau) - f(\rho))(g(\tau) - g(\rho))\} \geq 0,$$

Which implies that

$$f(\tau)g(\tau) + f(\rho)g(\rho) \geq f(\tau)g(\rho) + f(\rho)g(\tau) \tag{11}$$

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Consider

$$F(x,\tau) = \frac{x^{-\alpha-\beta-2\mu}\tau^{\mu}(x-\tau)^{\alpha-1}}{\Gamma_q(\alpha)}\ {}_2F_1\left[\alpha+\beta+\mu,-\eta;1-\frac{t}{x}\right]f(t)dt$$
$$= \frac{\tau^{\mu}(x-\tau)^{\alpha-1}}{\Gamma_q(\alpha)x^{\alpha+\beta+2\mu}} \times \frac{\tau^{\mu}(\alpha+\beta+\mu)(-\eta)(x-\tau)^{\alpha}}{\Gamma_q(\alpha+1)x^{\alpha+\beta+2\mu}}$$
$$+\frac{\tau^{\mu}(\alpha+\beta+\mu)(\alpha+\beta+\mu+1)(x-\tau)^{\alpha-1}}{\Gamma_q(\alpha+2)} \times \frac{(x-\tau)^{\alpha+1}}{x^{\alpha+\beta+2\mu+2}} \quad (12)$$

Multiple both sides of (11) by (12) and integrating w.r.t τ from 0 to t, we get

$$I_q^{\alpha,\beta,\eta}\{f(x)g(x)\} + f(\rho)g(\rho)I_q^{\alpha,\beta,\eta}\{1\} \geq g(\rho)I_q^{\alpha,\beta,\eta}\{f(x)\} + f(\rho)I_q^{\alpha,\beta,\eta}\{g(x)\} \quad (13)$$

Next, multiplying both sides of (13) by $F(x,\rho)$, $(\rho \in (0,t))$, where $F(x,\rho)$ is given by (12) and integrating w.r.t. ρ from 0 to t, we get the result

$$I_q^{\alpha,\beta,\eta}\{f(x)g(x)\} \geq \frac{\Gamma_q(1-\beta)\Gamma_q(1+\alpha+\eta)}{\Gamma_q(1-\beta+\eta)}x^{\beta+\mu} \times I_q^{\alpha,\beta,\eta}\{f(x)\}I_q^{\alpha,\beta,\eta}\{g(x)\}$$

Theorem 4 *Let f and g be two synchronous functions on $(0,\infty)$ then*

$$\frac{\Gamma_q(1+\mu)\Gamma_q(1-\beta+\eta)}{\Gamma_q(1-\beta)\Gamma_q(1+\mu+\alpha+\eta)x^{\beta+\mu}}I_q^{\delta,\gamma,\theta,\zeta}\{f(x)g(x)\} + \frac{\Gamma_q(1+\theta)\Gamma_q(1-\delta+\zeta)}{\Gamma_q(1-\delta)\Gamma_q(1+\theta+\gamma+\zeta)x^{\beta+\mu}}I_q^{\alpha,\beta,\eta,\mu}\{f(x)g(x)\}$$
$$\geq I_q^{\alpha,\beta,\eta,\mu}\{f(x)\}I_q^{\delta,\gamma,\theta,\zeta}\{g(x)\} + I_q^{\delta,\gamma,\theta,\zeta}\{g(x)\}I_q^{\alpha,\beta,\eta,\mu}\{f(x)\}$$

For all $x > 0$, $\alpha > max0, -\beta-\mu$, $\gamma > Max(0,-\delta,-\theta)$, $\mu,\theta > -1$.

Proof: Let f and g be two synchronous functions, then using definition 1, for all τ, $\rho \in (0,t), t \geq 0$, we have

$$\{(f(\tau)-f(\rho))(g(\tau)-g(\rho))\} \geq 0,$$

Which implies that

$$f(\tau)g(\tau) + f(\rho)g(\rho) \geq f(\tau)g(\rho) + f(\rho)g(\tau) \quad (14)$$

Consider

$$F(x,\tau) = \frac{x^{-\alpha-\beta-2\mu}\tau^{\mu}(x-\tau)^{\alpha-1}}{\Gamma_q(\alpha)}\ {}_2F_1\left[\alpha+\beta+\mu,-\eta;1-\frac{t}{x}\right]f(t)dt$$
$$= \frac{\tau^{\mu}(x-\tau)^{\alpha-1}}{\Gamma_q(\alpha)x^{\alpha+\beta+2\mu}} \times \frac{\tau^{\mu}(\alpha+\beta+\mu)(-\eta)(x-\tau)^{\alpha}}{\Gamma_q(\alpha+1)x^{\alpha+\beta+2\mu}}$$
$$+\frac{\tau^{\mu}(\alpha+\beta+\mu)(\alpha+\beta+\mu+1)(x-\tau)^{\alpha-1}}{\Gamma_q(\alpha+2)} \times \frac{(x-\tau)^{\alpha+1}}{x^{\alpha+\beta+2\mu+2}} \quad (15)$$

Multiply both sides of equation (14) by F(x,τ) explained in (15) and integrating w.r.t. τ from 0 to t and using Definition 1, we get

$$I_q^{\alpha,\beta,\eta}\{f(x)g(x)\} + f(\rho)g(\rho)I_q^{\alpha,\beta,\eta}\{1\} \geq g(\rho)I_q^{\alpha,\beta,\eta}\{f(x)\} + f(\rho)I_q^{\alpha,\beta,\eta}\{g(x)\} \quad (16)$$

Next, multiplying both sides of (16) by $F(x,\rho)$, $(\rho \in (0,t))$, where $F(x,\rho)$ is given by (15) and integrating w.r.t. ρ from 0 to t, we get the result

$$\frac{\Gamma_q(1+\mu)\Gamma_q(1-\beta+\eta)}{\Gamma_q(1-\beta)\Gamma_q(1+\mu+\alpha+\eta)x^{\beta+\mu}} I_q^{\delta,\gamma,\theta,\zeta}\{f(x)g(x)\} + \frac{\Gamma_q(1+\theta)\Gamma_q(1-\delta+\zeta)}{\Gamma_q(1-\delta)\Gamma_q(1+\theta+\gamma+\zeta)x^{\beta+\mu}} I_q^{\alpha,\beta,\eta,\mu}\{f(x)g(x)\}$$

$$\geq I_q^{\alpha,\beta,\eta,\mu}\{f(x)\} I_q^{\delta,\gamma,\theta,\zeta}\{g(x)\} + I_q^{\delta,\gamma,\theta,\zeta}\{g(x)\} I_q^{\alpha,\beta,\eta,\mu}\{f(x)\}$$

4 Special Cases:

Here, we take a quick look at a few implications of the findings in the preceding section. The operator in definition (1) would instantly decrease to the thoroughly studied, Erdelyi-Kober, and Riemann-Liouville type fractional integral operators, respectively, according to Curiel and Galué [13], given by the relationships below (see also [14, 16])

Case I:

$$I_{0,x}^{\alpha,\beta,\eta}\{f(x)\} = I_{0,x}^{\alpha,\beta,\eta,0}\{f(x)\} = \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)} \int_0^x (x-\tau)^{\alpha-1} \, {}_2F_1\left[\alpha+\beta, -\eta; 1-\frac{\tau}{x}\right] f(\tau)d\tau,$$

Here we get , Erdelyi-Kober fractional operator.

$$= I_{0,x}^{\alpha,\eta}\{f(x)\} = I_{0,x}^{\alpha,0,\eta,0}\{f(x)\} = \frac{x^{-\alpha-\eta}}{\Gamma(\alpha)} \int_0^x (x-\tau)^{\alpha-1} \tau^{\eta} f(\tau)d\tau,$$

Case II:
Here we get, Riemann-Liouville fractional operator

$$= R^{\alpha}\{f(x)\} = I_x^{\alpha,-\alpha,\eta,0}\{f(x)\} = \frac{1}{\Gamma(\alpha)} \int_0^x (x-\tau)^{\alpha-1} f(\tau)d\tau,$$

Case II:
Putting $\gamma = \alpha$, $\delta = \beta$, $\zeta = \eta$ and $\theta = \mu$ in theorem 2 reduces into theorem 1.

$$I_{0,x}^{\alpha,\beta,\eta,\mu}\{f(x)g(x)\} + I_{0,x}^{\delta,\gamma,\theta,\zeta}\{g(x)f(x)\} \geq I_{0,x}^{\alpha,\beta,\eta,\mu}\{f(x)\} I_{0,x}^{\delta,\gamma,\theta,\zeta}\{g(x)\} + I_{0,x}^{\delta,\gamma,\theta,\zeta}\{g(x)\} I_{0,x}^{\alpha,\beta,\eta}\{f(x)\}$$

5 Conclusion

Since the beginning of differential calculus, fractional calculus, or FC, has been used in mathematical theory. However, in the past 20 years, chaos has grown and FC's use has been apparent, demonstrating very little connection to FC ideals. FC has seen recent achievement in the scientific and engineering fields. Many scientific fields are seeing an increase in the profitability of research subjects. Now concentrate on the FC ideas. There has been some progress in the theory of dynamic systems, despite the shortcomings of the suggested models. We are currently in the early phases of developing these algorithms. Numerous case studies involving FC-based models were published. The research that illustrates the benefits of using FC theory in a variety of scientific and engineering domains.

Conflict of Interest: The authors declare that there is no conflict of interest.

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